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II. **Solution by J. SCHEFFER, Hagerstown, Md.**

The problem reduces to finding the number of products of n quantities and their power. By expanding, $\frac{1}{1-x_1x}$, $\frac{1}{1-x_2x}$, $\frac{1}{1-x_3x}$, etc., into series, we find,

$$\frac{1}{1-x_ix}=1+x_ix+x_i^2x^2+x_i^3x^3+\dots\dots\quad (i=1,\ 2,\ \dots\dots,\ n).$$

$\left(\frac{1}{1-x_1x}\right)\left(\frac{1}{1-x_2x}\right)\left(\frac{1}{1-x_3x}\right)\dots\dots=1+s_1x+s_2x^2+s_3x^3+\dots\dots$ where $s_1=x_1+x_2+\dots\dots x_n$, $s_2=x_1^2+x_2^2+x_3^2\dots\dots+x_1x_2+x_1x_3\dots\dots+x_2x_3\dots\dots$, $s_3=x_1^3+x_2^3+\dots\dots x_n^2$ $x_1^2x_2+x_1^2x_3+\dots\dots+x_1x_2x_3+\dots\dots$

To find the number of products in any of these sets of products, we put $x_1=x_2=x_3=\dots\dots=x_n=1$, and we get

$$\frac{1}{1-x_1x}\cdot\frac{1}{1-x_2x}\cdot\frac{1}{1-x_0x}\dots\dots=\frac{1}{(1-x)^n}=(1-x)^{-n}.$$

$\therefore s_n$ is equal to the coefficients of x^n in the expression of $(1-x)^{-n}$.

$\therefore s_n=\frac{n(n+1)(n+2)\dots\dots[n+(n-1)]}{n!}$, or, multiplying numerator and denominator by $(n-1)!$ we get $\frac{(2n-1)!}{(n-1)!n!}$ as the required result.

Also solved by L. E. Newcomb, and Elmer Schuyler.

GEOMETRY.

Two solutions of 238 were received from L. E. Newcomb, Los Gatos, Cal.

239. **Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.**

Divide the sides of a triangle ABC internally in P, Q, R, so that $BP/PC=CQ/QA=AR/RB$. QR cuts BC externally in S. Show that BS is to CS in the duplicate ratio of CP to PB.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Draw SD parallel to AB; let m=any proper fraction, and let $PC=ma$, $QA=mb$, $RB=mc$. Then

$$\frac{BP}{PC}=\frac{a(1-m)}{am}=\frac{CQ}{QA}=\frac{b(1-m)}{bm}=\frac{AR}{RB}=\frac{c(1-m)}{cm}=\frac{1-m}{m}.$$

Considering AB, AC as the axes of coördinates the equation to RQ is

$$\frac{x}{bm}+\frac{y}{c(1-m)}=1,\ \text{equation to } BC \text{ is } \frac{x}{b}+\frac{y}{c}=1.$$

$$\therefore x=AD=\frac{bm^2}{2m-1},\ y=DS=\frac{c(1-m)^2}{2m-1}=cn.$$

$$CD=AD-b=\frac{b(1-m)^2}{2m-1}=bn.$$

$CS=n\sqrt{(b^2+c^2-2bc\cos A)}=an$ since $\angle CDS=A$.
$BS=a+CS=a(n+1)$.

$$\therefore CS=\frac{a(1-m)^2}{2m-1},\ BS=\frac{am^2}{2m-1}.\quad \therefore BS:CS=m^2:(1-m)^2.$$

$CP^2=a^2m^2$, $BP^2=a^2(1-m)^2$.
$\therefore CP^2:BP^2=m^2:(1-m)^2$, $\therefore BS:CS=CP^2:BP^2$.

II. Solution by **G. W. GREENWOOD, M. A.** (Oxon), Lebanon, Ill.

Through C draw a parallel to RQ which will intersect AB in a point M, say, then

$$\frac{BS}{CS}=\frac{RB}{MR}.\ \text{But}\ \frac{MR}{CQ}=\frac{AR}{QA}.\ \therefore \frac{BS}{CS}=\frac{RB}{AR}.\frac{QA}{CQ}=\left(\frac{PC}{PB}\right)^2.$$

III. Solution by **J. SCHEFFER**, Hagerstown, Md.

Draw CF parallel to AB, meeting RQ in T.

From $\dfrac{PB}{PC}=\dfrac{QC}{QA}=\dfrac{RA}{RB}$, we have $PB.QA=PC.QC$, $PB.RB=PC.RA$.

By multiplication, $PB^2.QA.RB=PC^2.QC.RA$, or $\dfrac{PB^2}{PC^2}=\dfrac{QC.RA}{QA.RB}$ ..... (1).

But $\dfrac{RA}{CT}=\dfrac{QA}{QC}$, and $\dfrac{CT}{RB}=\dfrac{CS}{BS}$; $\therefore \dfrac{RA}{RB}=\dfrac{QA}{QC}.\dfrac{CS}{BS}$.

Substituting in (1), $\dfrac{PB^2}{PC^2}=\dfrac{CS}{BS}$.

240. Proposed by **B. F. BURLESON.**

The points P_1, P_2, P_3 in the perpendicular BD of an isosceles triangle with equal angles at A and C are at the intersection of the three perpendiculars of the triangles, the center of its inscribed, and the center of its circumscribed circles, respectively. The distance from P_1 to P_2 is $m=16$ rods, and the distance from P_2 to P_3 is $n=17$ rods. Required the radii R and r of the triangle's circumscribed and inscribed circles, the perpendiculars $BD=P_b$, the base $AC=2b$, and one of the equal sides as $AB=a$.

I. Solution by the **PROPOSER.**

We derive from Chauvenet's *Trigonometry*, equation 300 and equation 298,

$$R^2-2Rr=n^2\ldots\ldots\ldots(1),$$
$$R^2-4r^2=(m+n)^2-2m^2\cdots\cdots\cdots(2).$$

Resolving (1) and (2), we find

$$R=\frac{n^2}{n-m}=289 \text{ rods, and } r=\frac{m(2n-m)}{2(n-m)}=144 \text{ rods.}$$

We have in any isosceles triangle

$$P_b=\frac{2rb^2}{b^2-r^2}=\frac{4n^2-m^2}{2(n-m)}=450 \text{ rods}\ldots\ldots(3). \quad \text{Also } P_b=\sqrt{[(R^2-b^2)+R]}\ldots\ldots(4).$$

Equating (3) and (4) and solving, we find that

$$2b=AC=2\sqrt{\{r[2R-r-2\sqrt{(R^2-2Rr)}]\}}=\frac{n}{n-m}\sqrt{(4n^2-m^2)}=480 \text{ rods.}$$

Whence $a=AB=\sqrt{(P_b{}^2+b^2)}=\frac{n}{n-m}\sqrt{(4n^2-m^2)}=510$ rods.

II. Solution by A. H. HOLMES, Brunswick, Me.

We have $BD=\sqrt{(a^2-b^2)}$, $r=\frac{b\sqrt{(a^2-b^2)}}{a+b}$, and $R=\frac{a^2}{2\sqrt{(a^2-b^2)}}$,

$$DP_1=\frac{b^2}{\sqrt{(a^2-b^2)}},\ DP_2=r, \text{ and } DP_3=\frac{a^2-2b^2}{2\sqrt{(a^2-b^2)}}.$$

$$\therefore \frac{b\sqrt{(a^2-b^2)}}{a+b}-\frac{b^2}{\sqrt{(a^2-b^2)}}=m=16\ldots\ldots\ldots(1),$$

$$\frac{a^2-2b^2}{2\sqrt{(a^2-b^2)}}-\frac{b\sqrt{(a^2-b^2)}}{a+b}=n=17\ldots\ldots\ldots(2).$$

From (1), $\sqrt{(a^2-b^2)}=\frac{b}{m}(a-2b)$.

From (2), $\sqrt{(a^2-b^2)}=\frac{a}{2n}(a-2b)$. $\therefore a=\frac{2n}{m}b$.

Putting the value of a in (1), $b=\frac{m}{2(n-m)}\sqrt{(4n^2-m^2)}$.

$$\therefore a=\frac{n}{n-m}\sqrt{(4n^2-m^2)},\ BD=\frac{4n^2-m^2}{2(n-m)},\ r=\frac{m\,(2n-m)}{2(n-m)},\ R=\frac{n^2}{n-m}.$$

When $m=16$ and $n=17$, the equal sides become 510, base=480, perpendicular=450, $r=144$, $R=289$.

Also solved by G. B. M. Zerr, J. Scheffer, L. E. Newcomb.

241. Proposed by Editor EPSTEEN.

If two conics have each double contact with a third, their chords of contact with that conic, and two of the lines through their common points, will meet in a point and form a harmonic pencil.

I. Solution by F. D. POSEY, A. B., San Mateo, Cal.

Let $S=0$ be the equation of the conic which has double contact with the first conic in the line $\alpha_1=0$ and with the second conic in $\alpha_2=0$. The equation of these two conics may then be written

$$c-\lambda_1^2\alpha_1^2=0\,\ldots\ldots\ldots(1),\quad c-\lambda_2^2\alpha_2^2=0\,\ldots\ldots\ldots(2).$$

The two straight lines $\lambda_1^2\alpha_1^2-\lambda_2^2\alpha_2^2=0$ go through the common points of (1) and (2), through the intersections of $\alpha_1=0$ with $\alpha_2=0$, and it is evident that the lines $\alpha_1=0$, $\lambda_1\alpha_1-\lambda_2\alpha_2=0$, $\alpha_2=0$, $\lambda_1\alpha_1+\lambda_2\alpha_2=0$, form a harmonic pencil.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $S=0$ be the equation to the third conic. Then $S=k\alpha^2$, $S=k_1\beta^2$ are the equations to the other two conics (Salmon's *Conic Sections*, Art. 252, page 234). Subtracting, $\alpha^2-\beta^2=0$ or $(\alpha-\beta)(\alpha+\beta)=0$. This represents a pair of chords of intersection passing through the intersection of the chords of contact $\alpha=0$, $\beta=0$. Also $\alpha=0$, $\beta=0$, $\alpha-\beta=0$, $\alpha+\beta=0$ form a harmonic pencil. (See Salmon's *Conic Sections*, Art. 263, page 242).

Also solved by G. W. Greenwood.

242. Proposed by the late MARCUS BAKER.

In a trapezoid $ABCD$, upper base $BC=a$, lower base $AD=b$, a line CP is drawn from vertex C to any point P in the base AD, such that $PD=mb$. The line CP intersects the diagonal BP in M and MN is drawn parallel to the bases meeting CD in N; then is $M.N=\dfrac{abm}{a+bm}$.

Solution by the PROPOSER, and G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

From the similar triangles DMN, DBC, we have

$$\frac{MN}{a}=\frac{MD}{BD}.$$

From the similar triangles MPD, MBC, we have

$$\frac{MD}{BD}=\frac{mb}{a+mb}.\qquad\therefore MN=\frac{amb}{a+mb}.$$

Also solved by A. H. Holmes, G. B. M. Zerr, F. D. Posey, J. Scheffer, and L. E. Newcomb.